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SONYA KOVALEVSKY: THE RUSSIAN QUEEN OF POST-ENLIGHTENMENT MATHEMATICS

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Resumo

A ideia deste artigo é divulgar os principais aspectos da vida e da obra de Sonya Kovalevsky, como, por exemplo, como começou sua excelente disposição para aprender ciência e como superou as barreiras da época para poder prosseguir seus estudos científicos no exterior e realizer seu sonho de ocupar uma posição em uma universidade europeia. Para atingir meu objetivo, foi realizado um procedimento reflexivo e sistemático, no qual os dados foram obtidos por documentação indireta, isto é, pesquisa de uma bibliografia já existente, visto que busca narrar e explicar a história de uma grande personagem da Matemática do século XIX, pouco conhecida do público em geral. Trata-se, pois, de um artigo de compilação, lembrando que esse tipo de estudo consiste em reunir ordenadamente a bibliografia selecionada, combiná-la e dela extrair aquilo que se faz interessante para o cumprimento do objetivo. Como resultado, obtive um panorama da vida e da obra de Sonya Kovalevsky, desde sua infância até sua morte precoce em 1891, explorando sua capacidade matemática e talentos de escritora.

Palavras-chave: Sonya Kovalewsky; Vida; Obra.

Sonya Kovalevsky: A Rainha Russa da Matemática Pós-Iluminista

Abstract

The idea of this article is to disclose the main aspects of life and work of Sonya Kovalevsky, such as how she began her excellent disposition to learn science and how she overcame the barriers of the time to be able to follow scientific studies abroad and fulfill her dream of occupying a position in a European university. In order to reach my objective, a reflexive and systematic procedure was performed, in which the data were obtained through indirect documentation, that is, research of an existing bibliography, since it seeks to narrate and explain the history of a great mathematical character of the nineteenth century, little known of the public in general. It is, therefore, a compilation article, remembering that this type of study consists of orderly gathering the selected bibliography, combining it and extracting what is interesting for the accomplishment of the objective. As a result, I got an overview of the life and work of Sonya Kovalevsky, from her childhood to her early death in 1891, exploring her mathematic ability and writing talents.

Keywords: Sonya Kovalewsky; Life; Work.

Putting on perspective

The movement known as Enlightenment had its origins in the eighteenth-century France, but its end date is neither precise nor consensual among historians[1]. The term "Enlightenment" was related with clarification and illumination, since the "enlightened man" was essentially different from the man of Medieval and Renaissance societies, at least in the way of dealing with the knowledge of nature and his relation with God. Therefore, the eighteenth century became popularly known as the "century of lights".

According to the Enlightenment thinkers, man was a product of his environment, that is, the society and the education, which he acquired. In Medieval and Renaissance societies, education was under the yoke of the Church and the new thinkers who were growing throughout Europe in the eighteenth century frowned that upon. To those thinkers, the Church propagated an outdated philosophy, which was a barrier to the yearnings of the emerging progress. In general, medieval society was ignorant, fanatic and submissive to religious dogmas; and Renaissance society – despite having made a significant progress in the Mathematics field, especially in comparison to the Middle Ages – was not able to unleash the chains of the old concepts. Besides that, in the eighteenth century the development of Mathematics was mostly attached to the development of Physics. That was precisely why the education needed radical and imperative changes.

In this sense, the Enlightenment thinkers defended that reason should be at the vanguard on education. Through reason, the age of Enlightenment reveled a new way of thinking, for the main goal of the Enlightenment philosophers was the unceasing pursuit of happiness and by crossing that path they fought against injustice, religious intolerance and, above all, the concentration of privileges in the hands of a few rich and powerful. Among those privileges was the education.

As determined by the Enlightenment thinkers, reason was substantially important to the studies of emerging natural and social phenomena. In this sense, it is worth noting that the Enlightenment philosophers were deists, that is, they believed in a God that acted indirectly on men, through the laws of nature. Consequently, man can find himself and God within reason and science. Besides that, in nature, people would be essentially good, therefore the problems and the social inequalities were created and imposed by man himself, according to the organization of the society in force at the time. It follows that, to fix such situation, man would have to completely change the society around him, establishing the guarantees of the natural rights of individuals, as for example freedom and free possession of goods.

Likewise, in the eighteenth century, a period of economic transformations had begun which would deeply mark the society: the Industrial Revolution. Broadly speaking, this episode in man's history consisted in the redefinition of methods of production through incorporation of technological advancements, supported by scientific discoveries, together with the escalation of trading and evolution of transportation. The Industrial Revolution had its origins in England and spread throughout the nineteenth century to other countries in Europe, causing increase of income and the advent of large cities.

New technologies transformed many areas of production and, among them, mainly the production of paper. This fact promoted directly the printing industry, whose technical advancements caused a considerable cost reduction on printing material. Consequently, the possibility of broad access to books and newspapers arose. The triumph of technology, which the Industrial Revolution generated valued Mathematics as a fundamental tool to the maintenance, and expansion of industrialization, setting this discipline in a position of greater highlight than it had previously in Renaissance.

Still in the eighteenth century, the governments started acting systematically in the financing of science, as the Maecenas from Renaissance. Due to those initiatives, the scientific community began to organize itself by creating the academies. It is worth noting that, yet in the eighteenth century, the Royal Society of London and the *Académie des Sciences* of Paris had already been created. Later, already in the eighteenth century, the Berlin Academy and the Petersburg Academy were created. The science academies started having an active role in the society of the Enlightenment, promoting the cooperation and propagation of the History of Mathematics. Many scientific magazines were created and rapidly became the main vehicles to propagate mathematical ideas.

In the nineteenth century, new magazines exclusively dedicated to Mathematics arose. As examples, the ones that stand out are the *Journal für die Reine und Angewandte Mathematik* from Berlin and the *Journal de Mathématiques Pures et Appliquées* from Paris. It is noteworthy that both periodicals are still being published to this day and keeping their places among the most prestigious mathematical publications between scholars. Moreover, multiple mathematical societies, nationals and locals, were founded such as the London Mathematical Society and the *Société Mathématique de France*. They also started publishing their own Mathematics Magazines. That said, it is observed that Mathematics for the first time gained autonomy in relation to the motivations brought by Physics. A significant number of results were achieved and, in parallel, there was not only an effort in the pursuit of greater rigor compared to the seventeenth century but also an expansion of current mathematical limits. Hence an improvement in the studies of Geometry Arithmetic and Algebra occurred, as well as the emerging of new Analysis and Topology. Unhesitatingly, it was from the eighteenth and nineteenth centuries that Mathematics evolved towards a specialization, which would be its trademark later in the twentieth century. In this context, a unique and brilliant mathematician arose, whose life and work will be presented from now on. (GONDIM; SAPUNARU, 2016; HUBERMAN, 2016; KANT; MENDELSSOHN, 2006).

Life and Work

Sonya Kovalevsky was born under an aristocratic, authoritarian and patriarchal Russian family in Moscow, but before she completed 6 years old, she moved to Palibino, in the countryside of Russia. It seems that she developed a taste for Mathematics owning to the stories she heard about her grandfather, an extraordinary military topographer, and her great-grandfather, who besides being a mathematician was also an astronomer. However, her uncle Piortr was her main influence, as she frequently heard him talk about quadrature and asymptote, and even though she did not understand that, she felt inspired by it. Thus, since an early age, as soon as she began her studies, Sonya showed great aptitude for Mathematics. It is said that during her childhood, Sonya would spend hours inside her bedroom studying the mathematical notes that she put by her on the walls.

By her adolescence, around 1863, she moved to Moscow with her sister Anyuta, for besides Mathematics, Sonya also had great literary talent. At that time, she wrote several poetries, literary and theater reviews, articles for journals and a promising tale entitled "The private professor", published in a popular magazine edited by Dostoyevsky, to whom Sonya was in love. However, Sonya's greatest passion was indeed Mathematics.

Taking the opportunity of her stay in Moscow, Sonya managed to convince her father to let her study Mathematics in the naval school with Aleksander Strannoliubsky.

However, when finishing her studies with Strannoliubsky, she was prevented from continuing to improve since Russian universities, more precisely at Moscow University, would not accept women and since she was single, her father did not permit her to travel to attend a university, outside of Russia, that accepted women. Therefore, Sonya quickly married by convenience with Vladimir

[1]For practical purposes, it is considered in this work that Sonya Kovalevsky is a Enlightenment mathematician.

Kovalevsky, a promising paleontologist.

Soon after the wedding, the Kovalevsky moved to Heidelberg, in Germany. In her own words, her desire was always "[…] to be able to live for my job, surrounded by those who are concerned with the same questions." (LEFFER, 1895, p.52).

In 1869, already at Heidelberg University, Sonya studied with many renowned mathematicians and scientists, as for example, Hermann von Helmholtz and Gustav Kirchhoff. Nevertheless, among them, stood out Leo Königsberger, since he had been a student of the respected Karl Weierstrass at Berlin University. Königsberger referred to Weierstrass in such praise and enthusiasm that, in the end, he aroused Sonya's interest in meeting him and perhaps also becoming his student.

When Sonya got to Berlin, she encountered the university closed to the possibility of accepting her as a student. Despite having impressed the professors at Heidelberg University, she did not achieve her admission, because alike Moscow University, Berlin University would not accept women as students. Sonya's deception is revealed in the following comment: "The capital of Prussia showed to be outdated, despite all my arguments and efforts, I could not obtain permission to attend the University of Berlin." (KOVALEWSKAYA, 1978, p.216). She then sought directly for Weierstrass since he had received many recommendations from his former student Königsberger about her. That was how Weierstrass accepted her as a private student in 1870 and Sonya quickly became his favorite pupil, or rather saying, in the mathematician's own words: "[…] his most gifted student […]". (MITTAG-LEFFLER, 1923, p.172). Sonya studied with Weierstrass for four years, a time equivalent to that of a regular university graduation. At that time, she also wrote three large works: the first, entitled "On the Theory of Partial Differential Equations", which became her doctoral dissertation; the second, "On the Reduction of a Definite Class of Abelian Integrals of the Third Range", one of Weierstrass' favorite topics; and, the third, a supplement of the survey on the rings of Saturn entitled "Supplementary Research and Observations on Laplace's Research on the Form of the Saturn Rings". About her work with Weierstrass, Sonya commented:

> These studies had the deepest possible influence throughout my mathematical career and determined, in a definitive and irrevocable way, the direction I should follow in my later scientific work: all my work was done exactly in the spirit of Weierstrass. (KOVALEWSKAYA, 1978, p.216).

Indeed, her doctoral dissertation deals with the "Cauchy-Kovalevsky theorem", which are nowadays studied in partial differential equations. In 1874, at the age of 24, Göttingen University granted Sonya the desired doctoral degree*, in absentia*, *summa cum lauda*, after a great effort from Weierstrass, and with the help of many other mathematicians, among them Emil du Bois-Reymond, her former professor. It also can be noted here that due to the excellence of her work, Sonya was released from oral examination.

Unfortunately, without the opportunity of working as a university professor in the universities of Germany for being a woman, she and her husband decided to return to Russia, where once again she was denied the chance to work, even with her valuable doctoral degree. During this period, she returned to Palibino, lived in Moscow and St. Petersburg, where she sought out Gösta Mittag-Leffler, a former pupil of Weierstrass, on his own recommendation. The impression caused by Sonya on Mittag-Leffler was powerful. In his own words:

> What interested me most in St. Petersburg was to meet Madame Kovalevsky. Today (February 10, 1876) I spent many hours with her. As a woman she is delicious. She is beautiful, and when she speaks her face lights up in an expression of feminine goodness and superior intelligence ... Her manner is simple and natural, without any trace of pedantry or affectation. No longer, in all respects, a 'lady of the great world'. As a scholar she is distinguished by a clarity and an unusual precision of expression, as well as by a singularly rapid conception. One can easily see the degree of depth to which she directed her studies, and I fully understand why Weierstrass regards her as the most talented of his disciples. (MITTAG-LEFFLER, 1923, p.172).

Besides Mittag-Leffler, Sonya also initiates a fruitful friendship with Anne Charlotte Leffler, Mittag-Leffler's sister, who would come to be her official biographer, with whom she wrote the piece *The Struggle for Happiness* in 1887. At the same time, the Kovalevsky's had their only daughter, Sophia Kovalevsky, who was nicknamed "Fufa" by Sonya.

Since she could not achieve a position in the Russian universities, Sonya decides to abandon Mathematics and resume her other passion: Literature. It is noteworthy that, in Sonya's writings, women's rights were constantly approached, mainly the ones related to university education. About this theme, Sonya argued: "[If I can teach], I can […] open the universities to women, which up until now have only been opened by special favor, a favor that can be denied at any given moment." (LEFFER, 1895, p.51).

Nonetheless, Mathematics always came first in Sonya's life and, concomitantly, the financial situation in the Kovalevsky family was not doing well due to bad business decisions conducted by her husband Vladimir. For this reason, she decided to return to Germany by herself, where she met Weierstrass, both initiating there a new project. Next, she traveled to Paris, but in 1883, Vladimir commits suicide because of the marital and financial problems. That same year, Sonya once again contacted her advisor, who, with the help of Mittag-Leffler, obtained her a position at Stockholm University. Thus, Sonya finally became a university professor, and from then on, her work and talent began to be recognized.

Finally, Sonya gained fame in the European mathematical scene. Consequently, mathematicians from around the world started writing and asking her to read their papers in order to get her opinion. Besides the professors with which Sonya studied directly, she also traded letters and met other great mathematicians such as Carl Rung, Émile Picard, Charles Hermite and also literary celebrities such as Eliot, Chekhov, Darwin and Huxley. At the time, she was also the editor of the paper *Acta Mathematica,* which is still published to this day. In 1888, Sonya reached the top of her carrier by winning the *Prix Bordin* from the French Academy of Sciences with the paper "On the Problem of the Rotation of a Solid Body about a Fixed Point", after having generalized previous works from great physicists and mathematicians, discovering a solution for Euler's equation for a solid body over a fixed point. Karen Rappaport explains Sonya´s paper:

> Prior to Sonya Kovalevsky's work, the only solutions for moving a rigid body over a fixed point were developed for the two cases in which the body is symmetrical. In the first case, developed by Euler, there are no external forces, and the center of mass is fixed inside the body. This is the case that describes the movement of the earth. In the second case, derived from Lagrange, the fixed point and the center of gravity are both on the axis of symmetry of the body. This case describes the top movement. Sofa Kovalevsky developed the first of the special soluble cases for an asymmetrical top. In this case, the center of mass is no longer on an axis in the body. She solved the problem by constructing coordinates explicitly as ultra-elliptic functions of time. Kovalevsky continued this work in two more articles on the movement of a rigid body. These two received awards from the Swedish Academy of Sciences in 1889. His later work on the subject was lost. (RAPPAPORT, 1981, p.569).

It is noteworthy that the award was offered by the French Academy of Sciences in a competition among other fifteen articles. The one developed by Sonya was so superior to the others that led Mittag-Leffler to make the following comment:

> On the problem of rotating a solid body around a fixed point, Acta Math. 12 (1889), pp. 177-232, was crowned at a solemn ceremony of the Academy of Sciences in Paris on December 24, 1888, with the Bordin prize, increased from 3000 to 5000 francs.

Sonya was then awarded by the Swedish Academy of Sciences and in 1889 achieved the much-desired recognition of the Russian academic circles by becoming a corresponding member of the Russian Academy of Sciences. However, despite all honors, no position as a university professor was offered. (AWM, 1983; OSEN, 1975).

The Russian Queen of Mathematics

Of all the mathematical contributions left by Sonya, the one that deserved a greater prominence was the one that gave her the title of doctor, the paper "On the Theory of Partial Differential Equations", where the "Cauchy-Kovalevsky Theorem" is found. On the development of this work, Weierstrass caught the attention of Sonya when he said: "[...] you see, my dear Sonya, that your observation (which seemed so simple to you) on the distinctive property of partial differential equations [...] was for me the starting point for some interesting and very revealing researches." (POLUBARINOVA-KOCHINA, 1978, p.235). From the point of view of Mathematical Physics, another interest of Sonya, the theorem applies to the study of phenomena that depend on one or more variables, such as the propagation of a sound wave in the air, where the disturbance depends on time and space, that is, the state of the wave can vary at different instants and positions along the means in which it propagates. It is worth noting the special interest that Sonya had in Mathematical Physics, considering Mittag-Leffler's analysis:

> Sonya from her stay in Heidelberg, where she met Helmholtz and Kirchhoff, developed a taste for physics-mathematics. This interest has increased over his particular studies with Weierstrass. She began to delve into Maxwell's works and asked Weierstrass for advice on how she should prepare her studies. The illustrious geometer [Weierstrass] became deeply interested in physics-mathematics [...]. (MITTAG-LEFFER, 1923, p.157).

From the mathematical perspective, the theorem guarantees the existence and uniqueness of a real or complex analytical solution of a system of nonlinear equations, when the data and equations are described by analytic functions. It is important to remember that an analytic function is one that can be developed by a convergent Taylor series and a nonlinear equation is another one in which the function or one of its derivatives has a degree greater than one. At the time, much was said about Sonya's doctoral thesis:

> [...] the envious have tried to make us believe that Sonya, in drafting her doctoral thesis, was not as independent as he wanted to be, and that she owed Weierstrass more than herself. Weierstrass's own words are now proof to the contrary. The proof that the differential equation is formally satisfied by a power series that does not converge to no system of values of independent variables, was one of the most original parts of the thesis and was at that time a discovery of high importance. (MITTAG-LEFFER, 1923, p.146).

Briefly, the "Cauchy-Kovalevsky Theorem" expatiates about the existence of solutions to a system of differential equations M in *n* dimensions when the coefficients are analytic functions and its proof is valid for analytic functions of real or complex variables. It is worth noting that the theorem guarantees the existence and uniqueness of the problem of initial value relative to a first-order ordinary differential equation and its demonstration is essentially based on power series for functions of several variables, hence the analyticity. (RAPPAPORT, 1981; AMM, 1981).

The End

From 1884 until her death in 1891, Sonya worked as a professor of Mathematics at Stockholm University. Sadly, when she was conquering all the recognition of her potential and showing the world that the fact that she was a female did not shrink her intellectual capacity, Sonya died of pneumonia at the age of 41. At that time, she was at the peak of her career and had already obtained the recognition of the great mathematicians of her time. It is worth noting that shortly before her death she wrote several novels: *Sisters Rajevsky*, which was never published; *A Russian Childhood* and *Nihilist Girl*, both autobiographical works published in 1890; and *Vera Vorontzoff,* published *post mortem* in 1892. All her novels received praise from literary critics, who were surprised by Sonya's versatility in elegantly moving between Mathematics and Literature. Her early death was undeniably a great loss. (OSEN, 1975).

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